Alpha Decay

Energetics of the Alpha Decay

Alpha decay is the spontaneous emission of an α-particle, namely the ${}^{4}_{2}He$ nucleus. The process can be described by the following formula:

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$

and appears in very few light nuclides and mostly in heavy nuclides.

The energy release in an α-emission can be calculated using the above equation:

$$Q = [m_N({}^{A}_{Z}X) - m_N({}^{A-4}_{Z-2}Y) - m_\alpha]c^2$$

where m_N represents the mass of the nucleus. Since it's more convenient to deal with atomic masses, we can rewrite the above equation as:

$$Q = [m_N({}^{A}_{Z}X) - m_N({}^{A-4}_{Z-2}Y) - m({}^{4}_{2}He)]c^2$$

where the electron masses are properly accounted for and the electronic binding energy has been ignored. This also assumes that the transitions are "between nuclear ground states."

If $Q < 0$ the process is endothermic and cannot occur spontaneously

If $Q > 0$ the process is exothermic and can occur spontaneously

In the latter case the energy is given up to the α-particle and the daughter nucleus in the form of kinetic energy.

Calculating Q using nuclear data:

$$Q = -[B({}^{A}_{Z}X) - B({}^{A-4}_{Z-2}Y) - B({}^{4}_{2}He)]$$

where the binding energy of ${}^{4}_{2}He$ is known to be 28.3 MeV. Alpha emission is energetically favorably while the emission of other light nuclei from a heavy nucleus is rather unlikely. This is because the binding energy of ${}^{4}_{2}He$ is anomalously large.

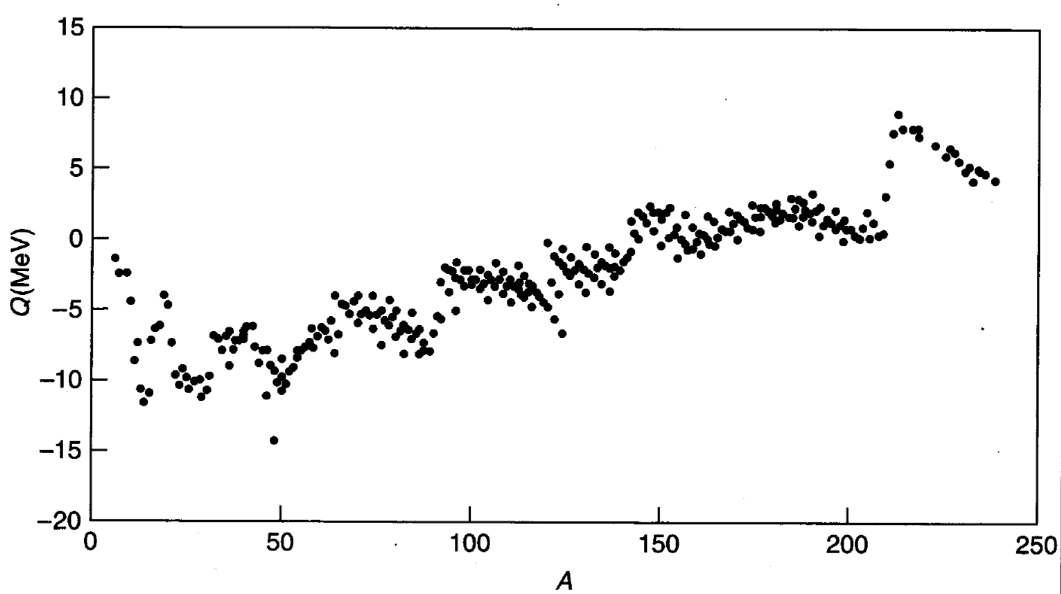
Table 8.1 | Energy, Q, Associated with the Emission of Various Particles form a ^{235}U Nucleus

Emitted Particle	Q (MeV)
n	−5.30
p	−6.70
^{2}H	−9.71
^{3}H	−9.97
^{3}He	−9.46
^{4}He	+4.68
^{6}Li	−3.85
^{7}Li	−2.88
^{7}Be	−3.79

For A less than about 150 the α-decay process is endothermic and does not occur; for A greater than ~150 the process becomes exothermic and can, in principle occur with a Q value that generally increases with increasing A.

Figure 8.1 | α-Decay Energies as a Function of A as Determined from Measured Atomic Masses

Data are shown for β-stable nuclides.



One of the key pieces of experimental information resulting from α-decays is measuring their kinetic energies. The Q value is split between (not equally) the α-particle and the daughter nucleus. Using conservation of momentum and energy, one can derive the following equation for the kinetic energy:

$$T_\alpha = \frac{Q}{1 + \frac{m_\alpha}{m_D}}$$

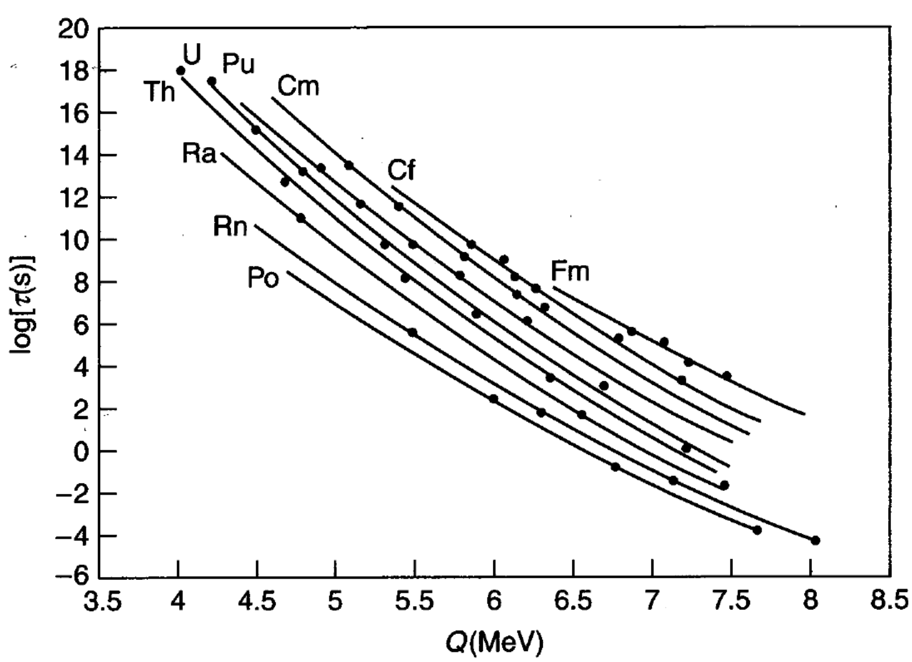
where m_D is the mass of the daughter nucleus. The energy recoil of the daughter nucleus accounts for about 2% of the total energy.

Geiger-Nuttall Relationship

The Geiger-Nuttall rule states that there is a dramtic decrease in the α-decay lifetime with increasing decay energy. This is shown in the following figure.

Figure 8.2 | Geiger-Nuttall Relationship Between the α-Decay Halflife and the Decay Energy for Some Even Z Nuclei

Each line represents data for a different value of Z as indicated by the element name.



Note: $^{232}_{90}Th \ (\tau = 6\times10^{17}\ s)$ and $^{218}_{90}Th \ (\tau = 1.4\times10^{-7}\ s)$

Theory of α-Decay

The basic theory of α-decay investigates the probability that two neutrons and two protons will become bound together within a nucleus, thereby creating an α-particle.

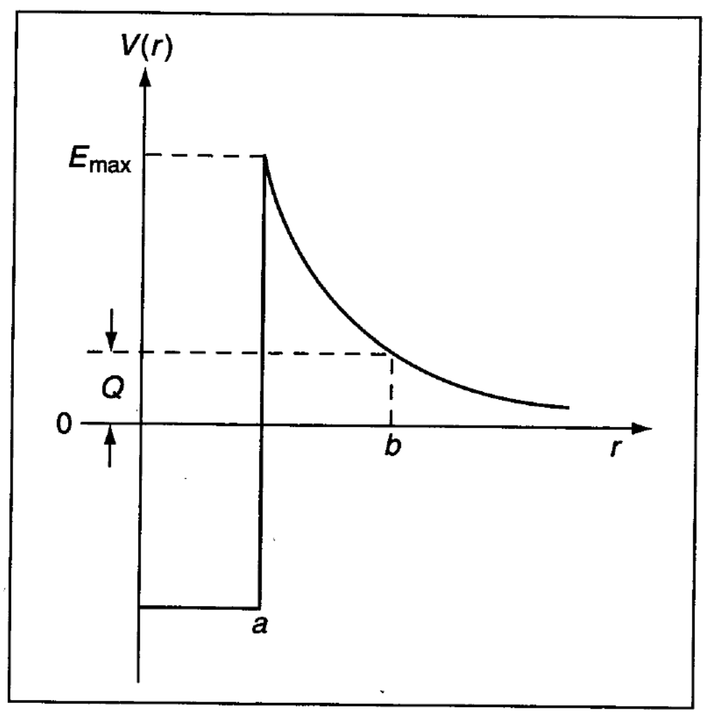
The lifetime for α-decay will then be given in terms of the:

1. time scale for α-particle formation within the nucleus, τ_0, and

2. the probability that the α-decay having been formed will escape from the nucleus, *P*.

$$\tau = \frac{\tau_0}{P}$$

Figure 8.3 | Potential Well for the α-Decay Model



Discussion of the Escape Probability

$$a = R_D + R_\alpha \qquad V(r) = \frac{2Z_D e^2}{4\pi\varepsilon_o r}$$

We can define two regions outside the nucleus:

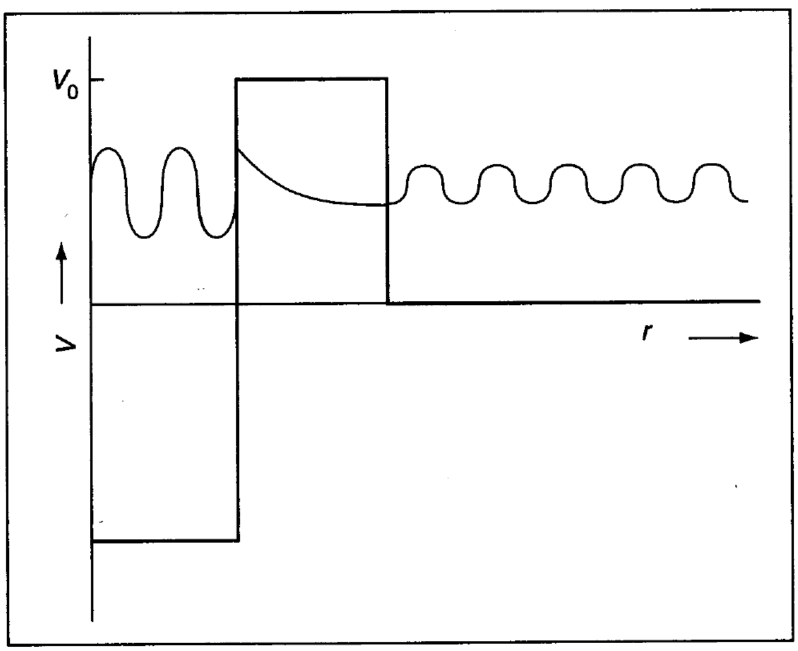
$$a < r < b$$

$$r > b$$

where

$$b = \frac{2Z_D e^2}{4\pi\varepsilon_o Q}$$

Figure 8.4 | Tunneling of an α-Particle Wave Function Through a Square Barrier



Probability

$$P = e^{-G}$$

where

$$G = \frac{4Z_D e^2}{4\pi\varepsilon_o}\sqrt{\frac{2m}{\hbar^2 Q}}\left[\cos^{-1}\sqrt{\frac{a}{b}} - \sqrt{\frac{a}{b}\left(1-\frac{a}{b}\right)}\right]$$

where m is the reduced mass of the α and the daughter nucleus.

The time scale τ_o depends on two factors. The first factor depends on the details of the processes that cause the formation of the α-particle within the nucleus. The second factor can be viewed classically as the time-interval between collisions of the α-particle off the nuclear potential well, and this is related to the velocity of the α-particle (which is related to the Q for the decay and the radius of the nucleus).

We don't expect that τ_o will be substantially different in nuclei with similar mass and similar nucleon configurations, so, we will assume that τ_o is a constant and write:

$$\tau = \frac{\tau_o}{e^{-G}}$$

Table 8.2 | Measured and Calculated α-Decay Lifetimes for Some Heavy Nuclei

Parent	Daughter	Q(MeV)	τ_{meas}(s)	τ_{calc}(s)
^{238}U	^{234}Th	4.27	2.0×10^{17}	3.0×10^{17}
^{234}U	^{230}Th	4.86	1.1×10^{13}	1.0×10^{13}
^{230}Th	^{226}Ra	4.77	3.5×10^{12}	3.5×10^{12}
^{226}Ra	^{222}Rn	4.87	7.4×10^{10}	6.6×10^{10}
^{222}Rn	^{218}Po	5.59	4.8×10^{5}	3.8×10^{5}
^{218}Po	^{214}Pb	6.11	2.6×10^{2}	1.4×10^{2}
^{214}Po	^{210}Pb	7.84	2.3×10^{-4}	1.0×10^{-4}
^{210}Po	^{206}Pb	5.41	1.7×10^{7}	5.2×10^{5}

The calculated lifetimes have been normalized to $^{230}_{90}Th$.

In summary:

1. The calculated lifetimes are ~ consistent with the measured lifetimes. $\tau_o = 6.3 \times 10^{-23}\ s$. This is determined from the $^{230}_{90}Th$ lifetime.

2. Provides a quantitative basis for the Geiger-Nutall rule.

3. When the α-emission is from an even-even nucleus, it is preferentially to the ground state of the daughter nucleus.